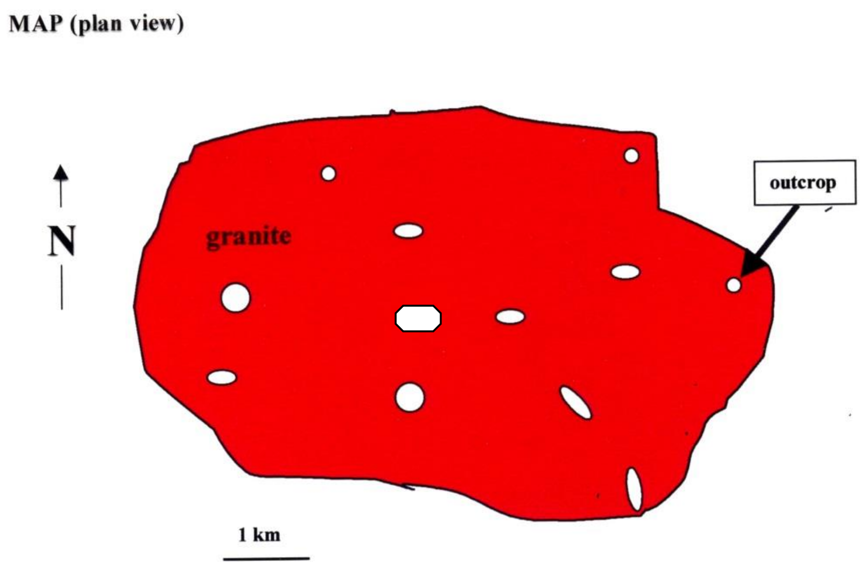
WB.ING-30

Numerical exercises

Ćwiczenia rachunkowe

Exercise 1

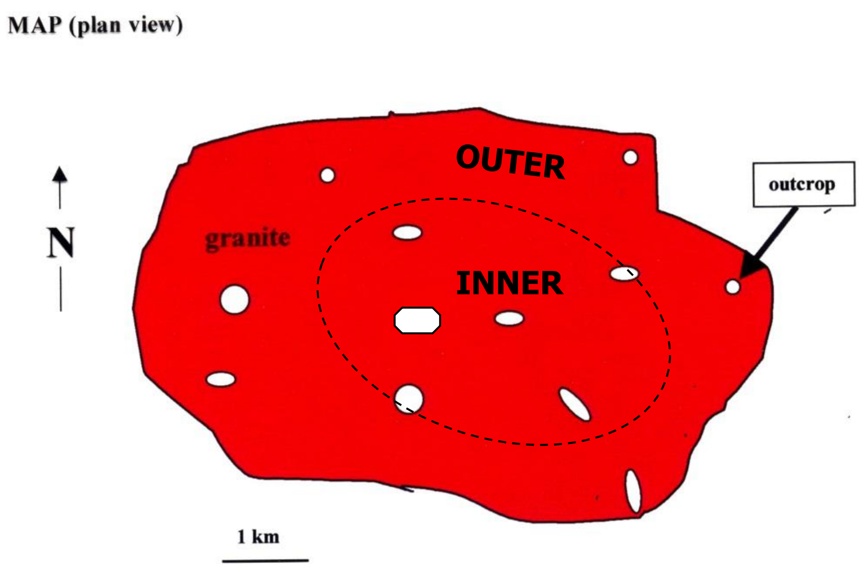


Alkalia ($K_2O+Na_2O+CaO+MgO$) wt.%

{X}

27.4
30.3
22.7
19.8
24.8
Mean: 25.0
27.4
30.3
Mean: 28.8
27.7
Mean: 26.8
19.8
24.8
Mean: 25.0
28.7
27.9
27.4
30.1
29.7
Mean: 27.4
Etc. etc.

Exercise 2



Alkalia (wt.%)

	INNER $\{X_1\}$ $n_1=5$	OUTER $\{X_2\}$ $n_2=5$
	27.4	23.0
	30.3	30.3
	22.7	26.4
	19.8	36.8
	24.8	33.5
Mean:	**25.0**	**30.0**

Exercise 3

$n_0 = 25$
$\bar{x}_0 = 36.5$ cm
$S_0 = 4.1$ cm
$S_a = 2.0$ cm
$\alpha = 0.10$ (10%)
$t_{1/2\alpha} = 1.711$ ← From Table 2 for DF = 24 and ½α = 0.05

$$n_{opt} = \frac{(t_{1/2\alpha} \cdot S_0/S_a)^2}{1 + \left(\frac{1}{n_0} \cdot t_{1/2\alpha} \cdot S_0/S_a\right)^2}$$

$$n_{opt} = \frac{(1.711 \cdot 4.1/2.0)^2}{1 + \left(\frac{1}{25} \cdot 1.711 \cdot 4.1/2.0\right)^2} = \frac{12.303}{1.019} \simeq \underline{\underline{13}}$$

Example 1

$n_0 = 25$
$\bar{x}_0 = 36.5$ cm
$S_0 = 4.1$ cm
$S_a = 1.0$ cm
$\alpha = 0.10$ (10%)

$$n_{opt} = \frac{\left(1.711 \, \frac{4.1}{1.0}\right)^2}{1 + \left(\frac{1}{25} \, 1.711 \, \frac{4.1}{1.0}\right)^2} = \frac{49.25}{1.08} \simeq \underline{\underline{46}}$$

$n = n_{opt} - n_0 = 46 - 25 = 21$ samples to be taken

Example 2

same as above, but with $\underline{S_0 = 14.1}$

$$n_{opt} = \frac{582.51}{1.93} \simeq \underline{\underline{302}}$$

$n = 302 - 25 = 277$ samples to be taken

Example 3

same as above, but with $\underline{S_0 = 2.1}$

$$n_{opt} = \frac{12.92}{1.02} \simeq \underline{\underline{13}}$$

no more samples needed

Example 4

same as above, but with $\underline{\alpha = 0.20}$ (20%)
$t_{1/2\alpha} = 1.318$

$$n_{opt} = \frac{29.2}{1.05} \simeq \underline{\underline{28}}$$

$n = 28 - 25 = 3$ samples to be taken

Example 5

same as above but with $\underline{\alpha = 0.05}$ (5%)
$t_{1/2\alpha} = 2.064$

$$n_{opt} = \frac{71.61}{1.11} \simeq \underline{\underline{65}}$$

$n = 65 - 25 = 40$ samples to be taken

Exercise 4

$n_0 = 25$
$\bar{x}_0 = 36.5$ cm
$s_0 = 4.1$ cm

$s_a = 2.0$ cm
$\alpha = 0.10$ (10%)
$t_{1/2\alpha} = 1.711$

Awashti & Kumar estimate:

$$\boxed{n_{opt} = \frac{(t_{1/2\alpha} \cdot s_0/s_a)^2}{1 + \left(\frac{1}{n_0} \cdot t_{1/2\alpha} \cdot s_0/s_a\right)^2}}$$

$$n_{opt} = \frac{(1.711 \cdot 4.1/2.0)^2}{1 + \left(\frac{1}{25} \cdot 1.711 \cdot 4.1/2.0\right)^2} = \frac{12.303}{1.019} \simeq \underline{\underline{13}}$$

Data as above

$L = 2.0$ cm
$\alpha = 0.10$ (10%)
$t_{1/2\alpha} = 1.711$

Estimate based on confidence interval:

$$\boxed{n_{opt} = \left(\frac{s_0 \cdot t_{1/2\alpha}}{L_\alpha}\right)^2}$$

$$n_{opt} = \left(\frac{4.1 \cdot 1.711}{2.0}\right)^2 \simeq \underline{\underline{13}}$$

Exercise 5

For grouped data (n-data, k-classes):

$$\boxed{\bar{x} = \frac{1}{n}\sum_{i=1}^{k} f_i x_i}$$

class midpoint value

class frequency

$$\boxed{s_x^2 = \frac{1}{n-1}\sum_{i=1}^{k} f_i (x_i - \bar{x})^2} \qquad \boxed{s_x = \sqrt{s_x^2}}$$

Example:

Classes	Midpoints x_i	Number frequencies f_i	$f_i x_i$	$f_i x_i^2$
0.0 – 1.0 cm	0.5 cm	4	2.0	1.00
1.0 – 2.0	1.5	14	21.0	31.50
2.0 – 3.0	2.5	38	95.0	237.50
3.0 – 4.0	3.5	42	147.0	514.50
4.0 – 5.0	4.5	23	103.5	465.75
5.0 – 6.0	5.5	6	33.0	181.50
k = 6		n = 127	401.5	1431.75

$$\bar{x} = \frac{401.5}{127} = 3.16 \simeq 3.2 \text{ cm}$$

$$s_x^2 = \frac{1}{n-1}\left[\sum^{k} f_i x_i^2 - \frac{\left(\sum^{k} f_i x_i\right)^2}{n}\right]$$

$$s_x^2 = \frac{1}{126}(1431.7 - 1269.3) = 1.29 \simeq 1.3 \text{ cm}^2$$

$$s_x = \sqrt{s_x^2} = 1.14 \simeq 1.1 \text{ cm}$$

Exercise 6

$\{x\}$ 16 22 21 20 23 21 19 15 13 23 17 20 29 18 22 16 25

$n = 17$

$\bar{x} = 20$

$S_x^2 = 14.9$

$S_{\bar{x}} = \sqrt{\frac{S_x^2}{n}} = \underline{0.93}$

For $\alpha = 0.05$, $t_{\frac{1}{2}\alpha = 0.025} = \underline{2.120}$

$DF = 16$

$L = \underline{2.120 \cdot 0.93} = 1.97 \simeq 2$

$18 < \mu < 22$ with 95% confidence

And if we wish to be 99% confident?

For $\alpha = 0.01$, $t_{\frac{1}{2}\alpha = 0.005} = 2.921$

$L = 2.921 \cdot 0.93 = 2.716 \simeq 3$

$17 < \mu < 23$ with confidence of 99%

Exercise 7

$\{X\}$
$n = 27$

$S_x^2 = 0.083 \qquad S_x = 0.288$

$$\sum^{n} (x - \bar{x})^2 = (n-1)\, S_x^2 = \underline{2.158}$$

For $\alpha = 0.10$

$$\chi^2_{\frac{1}{2}\alpha = 0.05} = \underline{38.90}$$

$$\chi^2_{0.95} = \underline{15.38}$$

$$\frac{2.158}{38.90} < \sigma^2 < \frac{2.158}{15.38}$$

$0.055 < \sigma^2 < 0.140$ with 90% confid.

$0.234 < \sigma < 0.374$ with 90% confid.

Exercise 8

Sandstone porosity (%)

$\{x_1\}$ $n_1 = 6$ $\{x_2\}$ $n_2 = 8$

x_1	x_2	x_1^2	x_2^2
13.9	14.7	193.2	216.1
12.5	12.1	156.2	146.4
11.0	15.2	121.0	231.0
11.8	13.6	139.2	184.9
10.8	11.5	116.6	132.2
14.6	15.4	213.1	237.1
	14.8		219.0
	13.7		187.7
$\sum x_1 = 74.6$	$\sum x_2 = 111.0$	$\sum x_1^2 = 939.3$	$\sum x_2^2 = 1554.4$

$$\bar{x}_1 = \frac{74.6}{6} = 12.4$$

$$\bar{x}_2 = \frac{111.0}{8} = 13.8$$

$$\boxed{s_x^2 = \frac{n\sum x^2 - (\sum x)^2}{n(n-1)}}$$

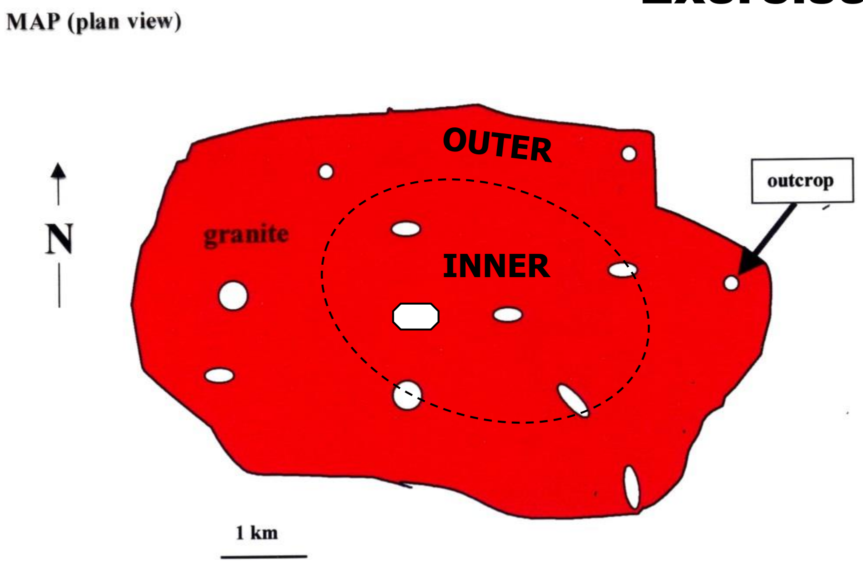
$$s_1^2 = \frac{6 \cdot 939.3 - (74.6)^2}{6(6-1)} = \frac{70.64}{30} = 2.35$$

$$s_1 = 1.5$$

$$s_2^2 = \frac{8 \cdot 1554.4 - (111.0)^2}{8(8-1)} = \frac{114.2}{56} = 2.04$$

$$s_2 = 1.4$$

Exercise 9



Alkalia (wt.%)

INNER $\{X_1\}$ $n_1=5$	OUTER $\{X_2\}$ $n_2=5$
27.6	28.3
23.8	33.5
22.9	27.2
28.2	31.1
22.5	29.9

Exercise 10

$\mu^* = 180$ assumed

$\{x\}$
$n = 16$

x	x^2
152	23104
184	33856
177	31329
199	·
178	·
180	·
186	·
183	·
194	·
185	·
179	·
184	·
186	·
215	·
246	·
198	39204
$\Sigma x = 3026$	$\Sigma x^2 = 578358$

H_0:

H_1:

$\bar{x} =$

$s_x^2 =$ $\qquad$ $s_x =$

$s_{\bar{x}} =$

$$t = \frac{|\bar{x} - \mu^*|}{s_{\bar{x}}} =$$

$t_{0.05}^{DF=15} =$

CONCLUSION:

Exercise 11

$\{X_1\}$ 57, 59, 59, 79, 81
$n_1 = 5$

$\left\{ \begin{array}{l} \sum x = 335 \\ \sum x^2 = 23013 \\ s_1^2 = 106 \end{array} \right.$

$\{X_2\}$ 72, 76, 83, 93, 86
$n_2 = 5$

$\left\{ \begin{array}{l} \sum x = 410 \\ \sum x^2 = 33894 \\ s_2^2 = 68 \end{array} \right.$

One-tail hypothesis to be tested:

$\left\{ \begin{array}{l} H_o: \\ H_1: \end{array} \right.$

The test F-function:

$F = s_1^2/s_2^2$

The calculated F-value

F =

The critical F-value:

for DF_1=
DF_2=
and $\alpha = 0.10$

$F_{0.10} =$

Result:

Conclusion:

Exercise 11 (cont.)

$\{X_1\}$ $n_1 = 5$	57, 59, 59, 79, 81	$\begin{cases} \sum x = 335 \\ \sum x^2 = 23013 \\ s_1^2 = 106 \end{cases}$
$\{X_2\}$ $n_2 = 5$	72, 76, 83, 93, 86	$\begin{cases} \sum x = 410 \\ \sum x^2 = 33894 \\ s_2^2 = 68 \end{cases}$

Mean: $\dot{x}_1 = 67$

Mean: $\dot{x}_2 = 82$

One-tail hypothesis to be tested:

$\begin{cases} H_o: \\ H_1: \end{cases}$

The test t-function:

t = … (see p. 27)

The calculated t-value:

t =

The critical t-value:

for DF = and $\alpha = 0.10$

$t_{0.10} =$

Result:

Conclusion:

Exercise 11 (cont.)

$\{X_1\}$ $n_1 = 5$ 57, 59, 59, 79, 81

$$\begin{cases} \sum x = 335 \\ \sum x^2 = 23013 \\ s_1^2 = 106 \end{cases}$$

Mean: $\dot{x}_1 = 67$

$\{X_2\}$ $n_2 = 5$ 72, 76, 83, 93, 86

$$\begin{cases} \sum x = 410 \\ \sum x^2 = 33894 \\ s_2^2 = 68 \end{cases}$$

Mean: $\dot{x}_2 = 82$

Exercise 12

The Cramer-von Mises normality test: an example

i	x_i	$x_i - \bar{x}$	$z_i = \frac{x_i - \bar{x}}{s_x}$	$P_i = P(z_i)$	$\frac{2i-1}{2n}$	$P_i - \frac{2i-1}{2n}$	$\left(P_i - \frac{2i-1}{2n}\right)^2$
1	16.01						
2	16.32						
3	17.30						
4	17.33						
5	17.60						
6	19.80						
7	24.31						
8	24.70						
9	26.50						
$n =$ 10	28.70						
						Sum:	

Critical value:

Exercise 13

The Pearson chi-square test: an example

Classes	n_i	x_i	$z_i = \frac{x_i - \bar{x}}{s_x}$	P_i	p_i	np_i	$(n_i - np_i)^2$	$\frac{(n_i - np_i)^2}{np_i}$
32.5 - 37.5	33	37.5						
37.5 - 42.5	93	42.5						
42.5 - 47.5	177	47.5						
47.5 - 52.5	266	52.5						
52.5 - 57.5	326	57.5						
57.5 - 62.5	290	62.5						
62.5 - 67.5	194	67.5						
67.5 - 72.5	91	72.5						
72.5 - 77.5	30	77.5						
$k=9$	$n=1500$							

$\chi^2_{0.05} =$

Exercise 14

The runs test for the difference between two populations – an example

Example: Turbidite sandstone thicknesses (cm) in two outcrops.

$\{X_1\}$ $n_1 = 10$: 28, 30, 24, 42, 33, 24, 31, 29, 35, 45

$\{X_2\}$ $n_2 = 9$: 27, 26, 34, 39, 25, 26, 27, 38, 43

Turbidite thicknesses (cm)

$U_o =$

$\bar{U}_e =$

$S^2_{\bar{u}} =$

$S_{\bar{u}} =$

Test function:

Find Z_α for $\alpha = 0.10$ (from Table 1B)

$Z_{0.10} =$ **Conclusion:**

Exercise 15

The Mann-Whitney (rank sum U) test for the difference between two populations – an example

$\{X_1\}$ $n_1 = 8$ 62, 63, 59, 54, 65, 60, 62, 57

$\{X_2\}$ $n_2 = 10$ 53, 58, 61, 62, 67, 52, 56, 58, 61, 63

Exercise 16

The Kolmogorov-Smirnov test: an example

Classes	$\{x_1\}$ Number frequency	Cumulative frequency	Cumul. freq.%	$\{x_2\}$ Number frequency	Cumulat. frequency	Cumul. freq. %	D
0 - 5	14			3			
5 - 10	5			1			
10 - 15	1			1			
15 - 20	10			9			
20 - 25	65			22			
25 - 30	229			41			
30 - 35	875			313			
35 - 40	802			679			
40 - 45	1324			871			
45 - 50	2167			3880			
	$n_1 = 5492$			$n_2 = 5820$			

Exercise 17

The chi-square test with contingency table – an example

	Species A	Species B	Species C
Bed 1	2	5	4
Bed 2	3	8	7

N =

Exercise 18

The chi-square test with contingency table – an example

	(1)	(2)	(3)	(4)	(5)	(6)	
$\{x_1\}$	6	26	65	50	17	2	=
$\{x_2\}$	6	18	70	60	45	8	=
							$N=$

Exercise 19

The Bartlett test – an example

$\{X_1\}$ $n_1=6$	$\{X_2\}$ $n_2=5$	$\{X_3\}$ $n_3=5$	$\{X_4\}$ $n_4=8$
48	42	33	78
49	39	42	69
67	51	46	60
75	57	47	52
53	75	50	63
33			45
			50
			35

$\Sigma x_i =$

$\bar{x}_i =$

Popul.	$(\Sigma x_i)^2$	Σx_i^2	DF	1/DF	S_i^2	$\log S_i^2$	$(n_i-1)\log S_i^2$
1							
2							
3							
4							
Sums:							

B =

χ^2 =

C =

Conclusion:

Exercise 20

$\{X_1\}$ $n_1=6$	$\{X_2\}$ $n_2=6$	$\{X_3\}$ $n_3=6$	$\{X_4\}$ $n_4=6$	$\{X_5\}$ $n_5=6$
19.2	18.7	12.5	20.3	19.9
18.7	14.3	14.3	22.5	24.3
21.3	20.2	8.7	17.6	17.6
16.5	17.6	11.4	18.4	20.2
17.3	19.3	9.5	15.9	18.4
22.4	16.1	16.5	19.0	19.1

$m=5$
$N=30$

$\sum^{n} X =$					
$(\sum^{n} X)^2 =$					
$\sum^{n} X^2 =$					
$\frac{(\sum^{n} X)^2}{n} =$					

$\sum^{m} \sum^{n} X =$	
$\sum^{m} (\sum^{n} X)^2 =$	
$\sum^{m} \sum^{n} X^2 =$	
$\sum^{m} \frac{(\sum^{n} X)^2}{n} =$	

Exercise 21

Factor 2

Factor 1	1	2	3
1	25	30	23
2	20	40	18
3	30	40	20
4	25	50	27

Exercise 22

$\{X_1\}$ $n_1=9$	$\{X_2\}$ $n_2=11$	$\{X_3\}$ $n_3=12$	$\{X_4\}$ $n_4=10$
10	8	8	18
11	12	3	17
12	12	4	16
12	13	9	12
14	16	12	18
8	9	6	17
6	3	7	15
9	7	7	8
10	15	9	10
	16	5	7
	15	6	
		10	

Value ranks

$$N = n_1 + n_2 + n_3 + n_4 = 42$$

Conclusion:

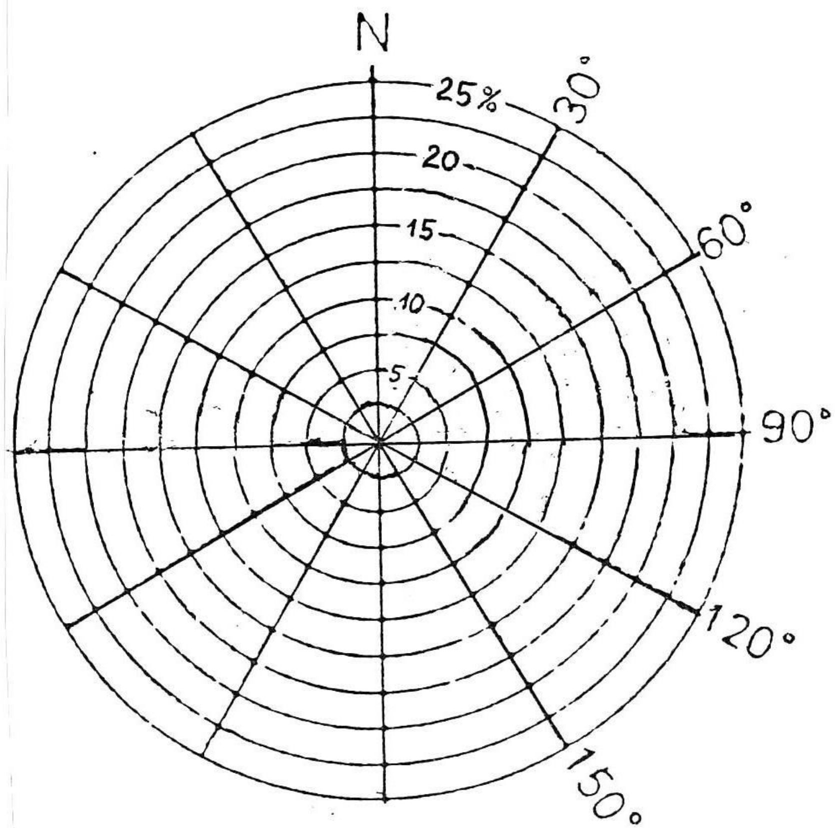
Exercise 23

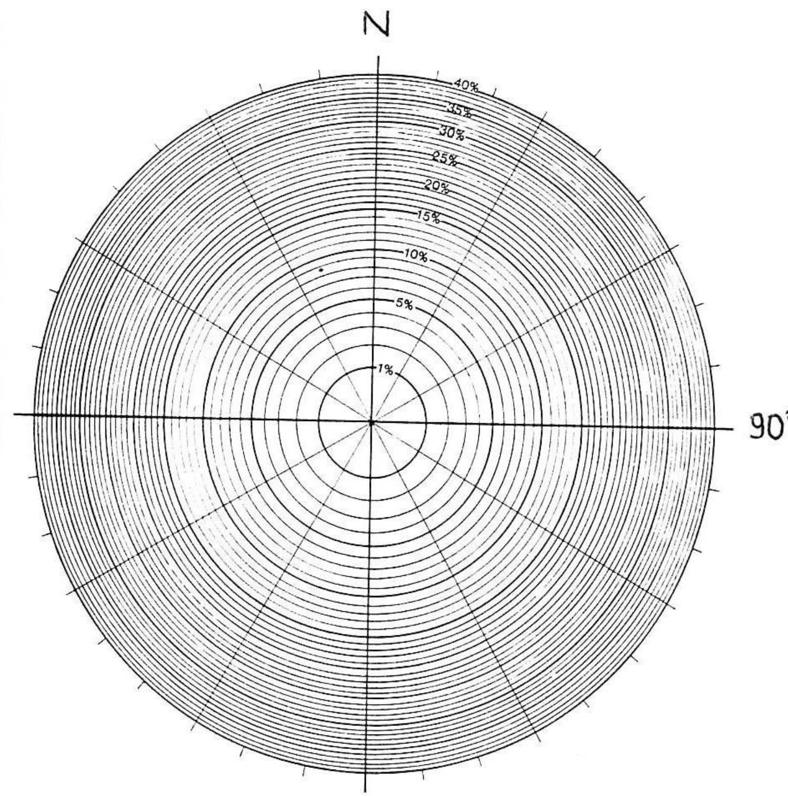
EXAMPLE

Measurements of the azimuth of ripple cross-laminae dip direction in a bay-fill sedimentary succession (Early Cretaceous Helvetiafjellet Fm., Spitsbergen). The data (n = 44 measurements) have been grouped in 30°-classes.

Azimuth class (degrees)	Frequency %
0 - 30	9.5
30 - 60	8.0
60 - 90	6.0
90 - 120	4.0
120 - 150	6.0
150 - 180	4.0
180 - 210	6.0
210 - 240	8.0
240 - 270	5.5
270 - 300	11.0
300 - 330	18.0
330 - 360	14.0

n = 44





Exercise 24

Example:

$N = 66$
$k = 6$

class	observed freq. (O_i)
0-60°	8
60-120°	9
120-180°	15
180-240°	20
240-300°	7
300-360°	7

Exercise 25

Grouped data

classes	x_i°	f_i %	$f_i \cos x_i^\circ$	$f_i \sin x_i^\circ$
0-30°	15	9.5	9.18	2.46
30-60°	45	8.0	5.66	5.66
60-90°	75	6.0	1.55	5.80
90-120°	105	4.0	-1.04	3.86
120-150°	135	6.0	-4.24	4.24
150-180°	165	4.0	-3.86	1.04
180-210°	195	6.0	-5.79	-1.55
210-240°	225	8.0	-5.66	-5.66
240-270°	255	5.5	-1.42	-5.31
270-300°	285	11.0	2.85	-10.63
300-330°	315	18.0	12.73	-12.73
330-360°	345	14.0	13.52	-3.62
sums:		100.0	23.48	-16.45
$n = 44$			$\bar{c} = 0.2348$	$\bar{s} = -0.1645$

Exercise 26

Calculation of the regression line

$$\begin{bmatrix} \Sigma y \\ \Sigma xy \end{bmatrix} = \begin{bmatrix} \Sigma x & n \\ \Sigma x^2 & \Sigma x \end{bmatrix} \begin{bmatrix} b \\ c \end{bmatrix}$$

$$\begin{bmatrix} 71.8 \\ 109.48 \end{bmatrix} = \begin{bmatrix} 15.1 & 10 \\ 23.23 & 15.1 \end{bmatrix} \begin{bmatrix} b \\ c \end{bmatrix}$$

$$d_* = \begin{vmatrix} 15.1 & 10 \\ 23.23 & 15.1 \end{vmatrix} = 15.1 \cdot 15.1 - 10 \cdot 23.23 = -4.29$$

$$d_1 = \begin{vmatrix} 71.8 & 10 \\ 109.48 & 15.1 \end{vmatrix} = 71.8 \cdot 15.1 - 10 \cdot 109.48 = -10.62$$

$$d_2 = \begin{vmatrix} 15.1 & 71.8 \\ 23.23 & 109.48 \end{vmatrix} = 15.1 \cdot 109.48 - 71.8 \cdot 23.23 = -14.76$$

$$b = \frac{d_1}{d_*} = \frac{-10.62}{-4.29} = 2.47$$

$$c = \frac{d_2}{d_*} = \frac{-14.76}{-4.29} = 3.44$$

$$\boxed{\hat{y} = 2.47\,x + 3.44}$$

$$\begin{cases} H_0: \beta \leq 0 \\ H_1: \beta > 0 \end{cases}$$

one-tail test

Fisher t-test

$$t = \frac{|b|\sqrt{\Sigma x^2 - n(\bar{x})^2}}{\sqrt{S_y^2(1 - r_{xy}^2)}} = \frac{2.47\sqrt{23.23 - 10(1.51)^2}}{\sqrt{0.579\,(1 - 0.417)}} =$$

$$= \frac{1.618}{0.581} = 2.785$$

$$\underset{(DF=8)}{t_{0.05}} = 1.860$$

Exercise 27

Calculation of residuals

$\{x, y\}_{n=10}$

	x	y	x^2	y^2	xy	$(xy)^2$	$\hat{y}$	$z^2 = (y-\hat{y})^2$
1	1.3	6.8	1.69	46.24	8.84	78.14	6.651	0.0222
2	1.4	7.3	1.96	53.29	10.22	104.44	6.898	0.1616
3	1.5	6.8	2.25	46.24	10.20	104.04	7.145	0.1190
4	1.4	6.2	1.96	38.44	8.68	75.34	6.898	0.4872
5	1.3	5.9	1.69	34.81	7.67	58.82	6.651	0.5640
6	1.3	7.7	1.69	59.29	10.01	100.20	6.651	1.1004
7	1.5	7.2	2.25	51.84	10.80	116.64	7.145	0.0030
8	1.7	7.9	2.89	62.41	13.43	180.36	7.639	0.0681
9	1.8	7.7	3.24	59.29	13.86	192.10	7.886	0.0346
10	1.9	8.3	3.61	68.89	15.77	248.69	8.133	0.0279
	$\sum x =$ 15.1	$\sum y =$ 71.8	$\sum x^2 =$ 23.23	$\sum y^2 =$ 520.74	$\sum xy =$ 109.48	$\sum (xy)^2 =$ 1258.80		$\sum z^2 =$ 2.5880

Exercise 28

The regression line's goodness-of-fit:

coefficient of determination

$$R^2 = r_{xy}^2(100\%) = 41.7\%$$

$$d^2 = \left(1 - \frac{s_z^2}{s_y^2}\right)100\% = \left(1 - \frac{0.287}{0.579}\right)100\% = 50.5\%$$

Confidence belt for regression line:

$$\begin{aligned}
x_k &= \bar{x} - 2s_x = 1.51 - 2(0.22) = \underline{1.07}\\
&= \bar{x} - s_x = 1.51 - 0.22 = \underline{1.29}\\
&= \bar{x} = \underline{1.51}\\
&= \bar{x} + s_x = 1.51 + 0.22 = \underline{1.73}\\
&= \bar{x} + 2s_x = 1.51 + 2(0.22) = \underline{1.95}
\end{aligned}$$

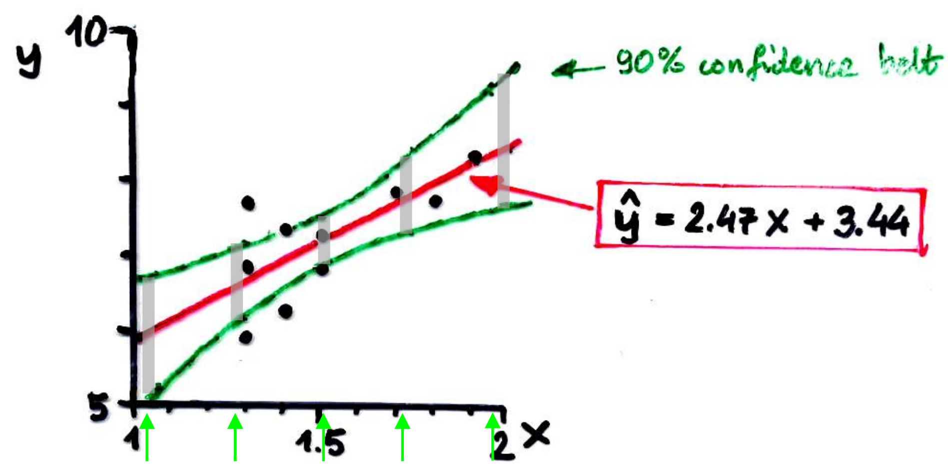
x_k	$\sqrt{R_k}$	$t_{1/2\alpha} \cdot s_z$	L_k
1.07	0.74	1.056	0.78
1.29	0.46	–ıı–	0.48
1.51	0.31	–ıı–	0.33
1.73	0.46	–ıı–	0.48
1.95	0.74	–ıı–	0.78

$$\boxed{L_k = t_{1/2\alpha} \cdot s_z \sqrt{R_k}}$$

$$\begin{aligned}
\sqrt{R_k} &= \sqrt{\frac{1}{10} + \frac{(1.07 - 1.51)^2}{(10-1)0.047}} = 0.74\\
&= \sqrt{\frac{1}{10} + \frac{(1.29 - 1.51)^2}{(10-1)0.047}} = 0.46\\
&= \sqrt{\frac{1}{10} + 0} = 0.31\\
&= \sqrt{\frac{1}{10} + \frac{(1.73 - 1.51)^2}{(10-1)0.047}} = 0.46\\
&= \sqrt{\frac{1}{10} + \frac{(1.95 - 1.51)^2}{(10-1)0.047}} = 0.74
\end{aligned}$$

$\alpha = 0.10$ $\qquad$ $t_{1/2\alpha} = 1.860$ (DF = 8) $\qquad$ $s_z = 0.568$

x_k	$\sqrt{R_k}$	$t_{1/2\alpha} \cdot s_z$	L_k
1.07	0.74	1.056	0.78
1.29	0.46	–ıı–	0.48
1.51	0.31	–ıı–	0.33
1.73	0.46	–ıı–	0.48
1.95	0.74	–ıı–	0.78

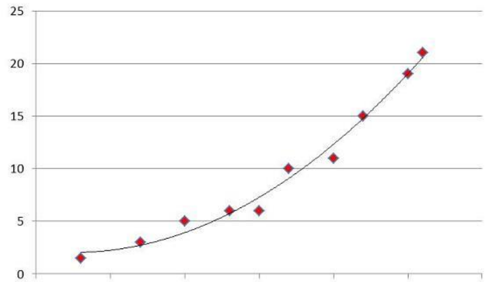


Exercise 29

2º polynomial (parabolic) model

$n = 10$

x	y	x^2	y^2	xy	x^2y	x^3	x^4
3	1.5						
7	3						
10	5						
13	6						
15	6						
17	10						
20	11						
22	15						
25	19						
26	21						
$\Sigma x =$ 158	$\Sigma y =$ 97.5	$\Sigma x^2 =$ 3026	$\Sigma y^2 =$ 1356.25	$\Sigma xy =$ 1984.5	$\Sigma x^2 y =$ 43645.5	$\Sigma x^3 =$ 63704	$\Sigma x^4 =$ 1417046



$\bar{x} = 15.8$ $\quad$ $\bar{y} = 9.75$

$S_x^2 = 58.84$ $\quad$ $S_y^2 = 45.07$

$S_x = 7.67$ $\quad$ $S_y = 6.71$

$S_{xy} = 44.40$ $\quad$ $S_{xy} = \overline{xy} - \bar{x}\cdot\bar{y} = \frac{1984.5}{10} - 15.8\cdot 9.75 =$

$r_{xy} = 0.863$ $\quad$ $r_{xy} = \frac{S_{xy}}{S_x \cdot S_y} = \frac{44.40}{7.67\cdot 6.71} =$

$$\boxed{\hat{y} = b_1 x + b_2 x^2 + c}$$

$$\begin{bmatrix} \sum_{}^{n} y_i \\ \sum_{}^{n} x_i y_i \\ \sum_{}^{n} x_i^2 y_i \end{bmatrix} = \begin{bmatrix} \sum^n x_i & \sum^n x_i^2 & n \\ \sum^n x_i^2 & \sum^n x_i^3 & \sum x_i \\ \sum^n x_i^3 & \sum^n x_i^4 & \sum x_i^2 \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ b_2 \\ c \end{bmatrix}$$

$$\begin{bmatrix} \Sigma y \\ \Sigma xy \\ \Sigma x^2 y \end{bmatrix} = \begin{bmatrix} \Sigma x & \Sigma x^2 & n \\ \Sigma x^2 & \Sigma x^3 & \Sigma x \\ \Sigma x^3 & \Sigma x^4 & \Sigma x^2 \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ b_2 \\ c \end{bmatrix}$$

$$\begin{bmatrix} 97.5 \\ 1984.5 \\ 43645.5 \end{bmatrix} = \begin{bmatrix} 158 & 3026 & 10 \\ 3026 & 63704 & 158 \\ 63704 & 1417046 & 3026 \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ b_2 \\ c \end{bmatrix}$$

$$d_* = \begin{vmatrix} 158 & 3026 & 10 \\ 3026 & 63704 & 158 \\ 63704 & 1417046 & 3026 \\ 158 & 3026 & 10 \\ 3026 & 63704 & 158 \end{vmatrix} =$$

$$d_1 = \begin{vmatrix} 97.5 & 3026 & 10 \\ 1984.5 & 63704 & 158 \\ 43645.5 & 1417046 & 3026 \\ 97.5 & 3026 & 10 \\ 1984.5 & 63704 & 158 \end{vmatrix} =$$

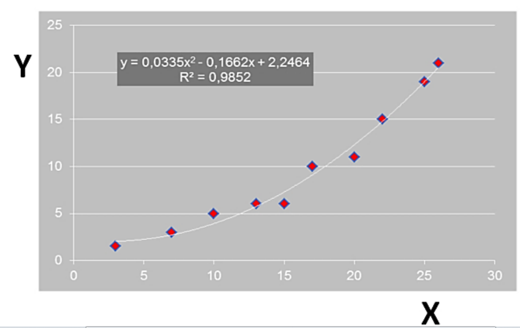
$$d_2 = \begin{vmatrix} 158 & 97.5 & 10 \\ 3026 & 1984.5 & 158 \\ 63704 & 43645.5 & 3026 \\ 158 & 97.5 & 10 \\ 3026 & 1984.5 & 158 \end{vmatrix} =$$

$$d_3 = \begin{vmatrix} 158 & 3026 & 97.5 \\ 3026 & 63704 & 1984.5 \\ 63704 & 1417046 & 43645.5 \\ 158 & 3026 & 97.5 \\ 3026 & 63704 & 1984.5 \end{vmatrix} =$$

$b_1 = \frac{d_1}{d_*}$ $\quad$ $b_2 = \frac{d_2}{d_*}$ $\quad$ $c = \frac{d_3}{d_*}$

$$\boxed{\hat{y} = b_1 x + b_2 x^2 + c}$$

$b_1 = -0.1662$ $\quad$ $b_2 = 0.0335$ $\quad$ $c = 2.2464$



Exercise 30

$n = 7$

x	y	x^2	y^2	xy	$\ln y$	$x \cdot \ln y$
1	2					
2	3					
3	5					
4	7					
5	10					
6	14					
7	26					
$\Sigma x = 28$	$\Sigma y = 67$	$\Sigma x^2 = 140$	$\Sigma y^2 = 1059$	$\Sigma xy = 367$	$\Sigma \ln y = 13.55$	$\Sigma x \ln y = 65.69$

$\bar{x} = 4$ $\qquad$ $\bar{y} = 9.57$

$S_x^2 = 4.66$ $\qquad$ $S_y^2 = 69.62$

$S_x = 2.16$ $\qquad$ $S_y = 8.34$

$S_{xy} = 16.5$

$r_{xy} = 0.92$

$$\begin{bmatrix} \Sigma \ln y \\ \Sigma x \ln y \end{bmatrix} = \begin{bmatrix} \Sigma x & n \\ \Sigma x^2 & \Sigma x \end{bmatrix} \cdot \begin{bmatrix} \ln b \\ \ln c \end{bmatrix} \begin{matrix} \leftarrow 0.41 \\ \leftarrow 0.29 \end{matrix}$$

$$\boxed{\hat{y} = c \cdot b^x}$$

$$\hat{y} = 1.34 \cdot 1.51^x$$

68

Exponential model: $\boxed{\hat{y} = b^x \cdot c}$

This has to be transformed into a linear form:

$$\ln y = x \cdot \ln b + \ln c$$

and the coefficients (b and c) are found by solving the following system of linear equations:

$$\begin{cases} \sum_{i=1}^{n} \ln y_i = \boxed{\ln b} \cdot \sum_{i=1}^{n} x_i + \boxed{\ln c} \cdot n \\ \sum_{i=1}^{n} x_i \cdot \ln y_i = \boxed{\ln b} \cdot \sum_{i=1}^{n} x_i^2 + \boxed{\ln c} \cdot \sum_{i=1}^{n} x_i \end{cases}$$

or in matrix form:

$$\begin{bmatrix} \sum_{i=1}^{n} \ln y_i \\ \sum_{i=1}^{n} x_i \ln y_i \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} x_i & n \\ \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i \end{bmatrix} \cdot \begin{bmatrix} \boxed{\ln b} \\ \boxed{\ln c} \end{bmatrix}$$

Exercise 31

$n=5$

x	y	x^2	y^2	xy	$\ln x$	$x\ln x$
0.5	2					
1	6					
2	10					
4	15					
8	17					
$\Sigma x=$ 15.5	$\Sigma y=$ 50	$\Sigma x^2=$ 82.25	$\Sigma y^2=$ 654	$\Sigma xy=$ 223	$\Sigma \ln x=$	$\Sigma x\ln x=$

Logarithmic model: $\boxed{\hat{y} = b\cdot \ln x + c}$

Here, the following system of linear equations has to be solved to find "b" and "c":

$$\begin{cases} \sum_{i=1}^{n} y_i = \boxed{b} \sum_{i=1}^{n} \ln x_i + \boxed{c}\cdot n \\ \sum_{i=1}^{n} x_i y_i = \boxed{b} \sum_{i=1}^{n} x_i \ln x_i + \boxed{c}\cdot \sum_{i=1}^{n} x_i \end{cases}$$

$\bar{x} = 3.1$ $\quad$ $\bar{y} = 10$

$S_x^2 = 9.3$ $\quad$ $S_y^2 = 38.5$

$S_x = 3.05$ $\quad$ $S_y = 6.20$

$r_{xy} = 0.899$

$$\begin{bmatrix} \Sigma y \\ \Sigma xy \end{bmatrix} = \begin{bmatrix} \Sigma \ln x & n \\ \Sigma x \ln x & \Sigma x \end{bmatrix} \cdot \begin{bmatrix} b \\ c \end{bmatrix}$$

$$\boxed{\hat{y} = b \cdot \ln x + c}$$

$$\hat{y} = 5.43 \ln x + 6.25$$

Exercise 32

$n=8$

x	y	x^2	y^2	xy	$\ln x$	$\ln y$	$x\ln x$	$x\ln y$
1	10							
1.5	25							
2	30							
3	35							
4	40							
10	50							
16	60							
23	70							
$\Sigma x =$ 60.5	$\Sigma y =$ 320	$\Sigma x^2 =$ 917.25	$\Sigma y^2 =$ 15450	$\Sigma xy =$ 3442.5	$\Sigma \ln x =$ 11.8	$\Sigma \ln y =$ 28.38	$\Sigma x\ln x =$ 150.53	$\Sigma x \ln y =$ 212.17

Hyperbolic (power-law) model

$$\boxed{\hat{y} = c \cdot x^b}$$

$\bar{x} = 7.56$ $\quad \bar{y} = 40$

$S_x^2 = 65.67$ $\quad S_y^2 = 378.57$

$S_x = 8.10$ $\quad S_y = 19.45$

$S_{xy} = 146.07$

$r_{xy} = 0.927$

$$\begin{bmatrix} \Sigma \ln y \\ \Sigma x \ln y \end{bmatrix} = \begin{bmatrix} \Sigma \ln x & n \\ \Sigma x \ln x & \Sigma x \end{bmatrix} \cdot \begin{bmatrix} b \\ \ln c \end{bmatrix}$$

$$\begin{bmatrix} 28.38 \\ 212.17 \end{bmatrix} = \begin{bmatrix} 11.8 & 8 \\ 150.53 & 60.5 \end{bmatrix} \cdot \begin{bmatrix} b \\ \ln c \end{bmatrix}$$

$$\boxed{\hat{y} = c \cdot x^b} \qquad \hat{y} = 17\, x^{0.489}$$

Exercise 33

$\{x, y\}$

$n = 10$

$D = R_x - R_y$

x	y	R_x	R_y	D	D^2
1.3	6.8	2	3.5	-1.5	2.25
1.4	7.3	4.5	6	-1.5	2.25
1.5	6.8	6.5	3.5	3.0	9.00
1.4	6.2	4.5	2	2.5	6.25
1.3	5.9	2	1	1	1.00
1.3	7.7	2	7.5	-5.5	30.25
1.5	7.2	6.5	5	1.5	2.25
1.7	7.9	8	9	-1	1.00
1.8	7.7	9	7.5	1.5	2.25
1.9	8.3	10	10	0	0.00

(Ranks: R_x, R_y)

$\sum D^2 = 56.5$

$$r_s = 1 - \frac{6 \cdot 56.5}{10(100-1)} = 0.657$$

$m_x = 7$

$m_y = 4$

$$\begin{cases} H_0: \rho_s \le 0 \\ H_1: \rho_s > 0 \end{cases}$$

$$t = 0.657\sqrt{\frac{10-2}{1-0.431}} = 2.465$$

$$\left.\begin{matrix} DF = 8 \\ \alpha = 0.05 \end{matrix}\right\} t_{0.05} = 1.860$$

Correction for rank ties:

$$\boxed{r_s = \frac{E_x + E_y - \sum^n d^2}{2\sqrt{E_x \cdot E_y}}}$$

$$E_x = \frac{10^3 - 10}{12} - \frac{7^3 - 7}{12} = 82.5 - 28 = 54.5$$

$$E_y = \frac{10^3 - 10}{12} - \frac{4^3 - 4}{12} = 82.5 - 5 = 77.5$$

$$r_s = \frac{54.5 + 77.5 - 56.5}{2\sqrt{54.5 \cdot 77.5}} = \frac{75.5}{129.9} = 0.581$$

$$\begin{cases} H_0: \rho_s \le 0 \\ H_1: \rho_s > 0 \end{cases}$$

$$t = 0.581\sqrt{\frac{10-2}{1-(0.581)^2}} = 2.019$$

$$t_{0.05} = 1.860$$

Exercise 34

$$b = \frac{0.8200 - 0.7707}{20.282 - 10.187} = \frac{0.0493}{10.095} = 0.00488$$

x	y	b	Z_i	D_i
10.187	0.7707		0.719561	0.000129
20.282	0.8200	0.00488	0.718184	0.001248
29.889	0.8706	0.00526	0.720557	0.001125
39.900	0.9196	0.00489	0.719302	0.000130
49.875	0.9704	0.00511	0.720028	0.000596
60.342	1.0192	0.00466	0.716283	0.003149
70.889	1.0704	0.00485	0.714537	0.004895
79.528	1.1207	0.00582	0.721469	0.002037
89.982	1.1702	0.00473	0.718490	0.000942
99.620	1.2202	0.00519	0.720108	0.000676
		etc.		

$N = n(n-1)/2$ local gradients for $1 \leq i < j \leq n$

$\boxed{D_m = 0.001034}$

$\boxed{b_m = 0.00502}$ $\boxed{Z_m = 0.719432}$

Limits for outliers

If larger than $\rightarrow$ and/or if smaller than $\rightarrow$

$$\begin{cases} Z_m + 3D_m = 0.72253 \\ Z_m - 3D_m = 0.71633 \end{cases}$$

Samples 6 and 7 are outliers; new $b_m = 0.00503$

Estimation of $c = 0.7194$ $(+0.0011, -0.0013)$

Exercise 35

Basal grain size of turbidite bed

Degree of bed bioturbation

	(1)	(2)	(3)	(4)	(5)	(6)	
(1)	18	0	1	0	0	0	19
(2)	5	4	0	0	0	0	9
(3)	4	1	0	0	0	0	5
(4)	2	2	0	0	0	0	4
(5)	0	1	1	0	0	0	2
(6)	0	0	0	0	0	0	0
	29	8	2	0	0	0	n=39

Exercise 36

			A	B	C	
Grey clayey lmst.	**1**	o_{ij}	**39**	**18**	**10**	**67**
		e_{ij}				
		d_{ij}				
Lmst & anhydr.	**2**	o_{ij}	**65**	**37**	**44**	**146**
		e_{ij}				
		d_{ij}				
White lmst	**3**	o_{ij}	**43**	**40**	**13**	**96**
		e_{ij}				
		d_{ij}				
			147	**95**	**67**	**309**

Exercise 36

Make your calculation notes here

Exercise 37

	Samples	OA forams	OWB forams	OWP forams	OWPk forams
1	So1	50	141	3	1
2	So2	280	18	1	0
3	So3	10	14	139	0
4	So4	90	24	7	0
5	So5	53	19	18	0
6	So6	250	34	13	2
7	So8	296	4	0	0
8	So9	90	0	0	0
9	So10	72	0	0	0
10	So11	240	0	0	0
11	So12	297	0	0	0
12	So14	294	6	0	0
13	So15	14	6	1	0
14	So17	57	5	0	0
15	So18	292	7	1	0
16	So19	7	1	0	0
17	So23	65	7	3	0
18	So24	5	110	23	0
19	So25	14	26	7	0
20	So26	39	0	0	0
21	So27	33	0	0	0
22	So28	27	2	0	0
23	So29	60	3	0	0
24	Rz1	25	0	0	0
25	Rz2	0	0	0	0
26	Rz4	0	84	0	1
27	Rz6	0	0	0	0
28	Rz7	0	0	0	0
29	Rz8	0	0	0	0
30	Rz9	0	0	0	0
31	Rz10	0	0	0	0
32	S-04	2	27	15	0

Affinity matrix

	OA	OWB	OWP	OWPk
OA	0			
OWB		0		
OWP			0	
OWPk				0

Pitagorean taxonomic distance d_{ij} used

Taxonomic distance d_{ij} for variables OA and OWB

$$d_{ij} = [(50-141)^2 + (280-18)^2 + (10-14)^2 + (90-24)^2 + \ldots (2-27)^2]^{1/2}$$

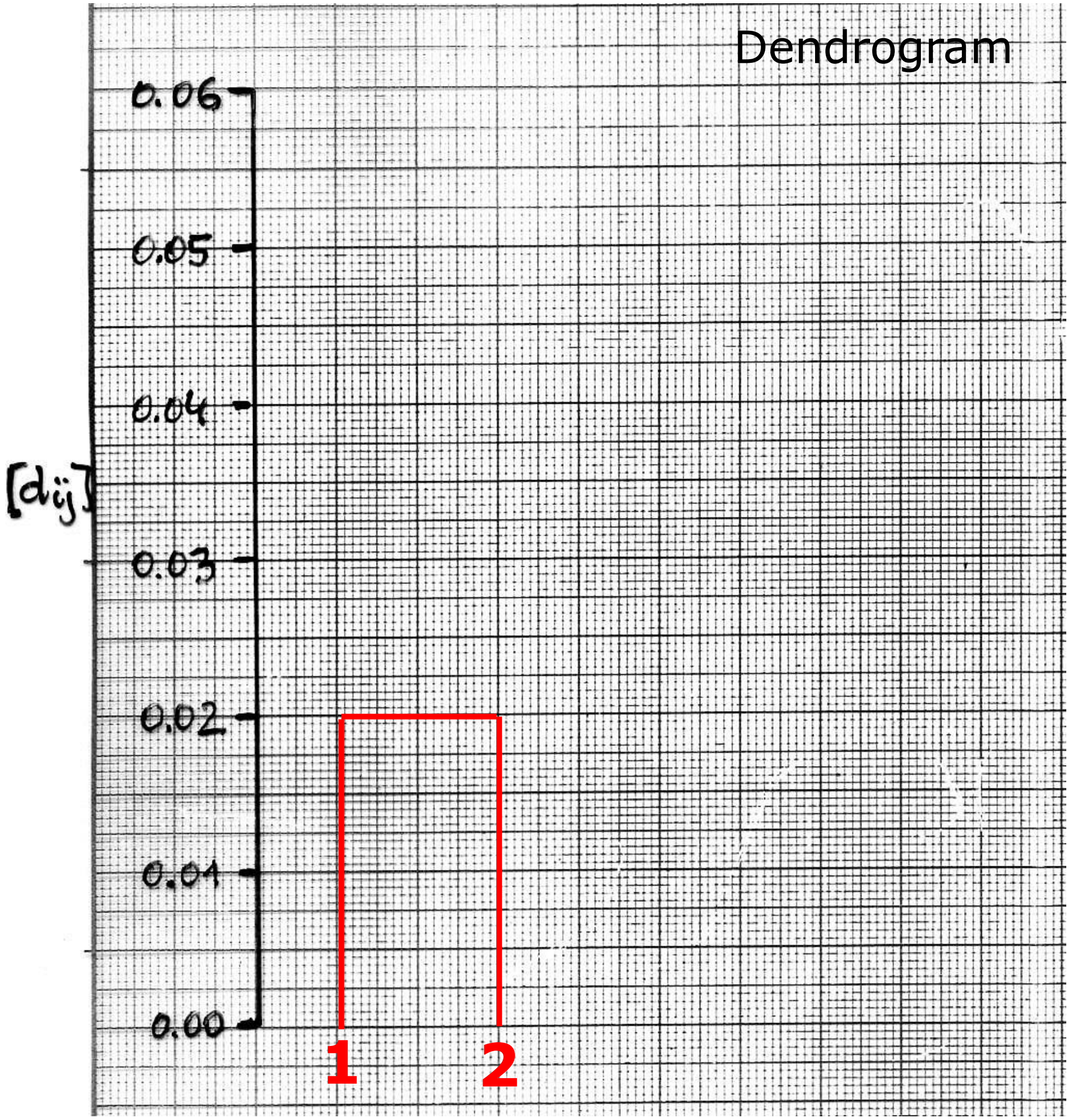
$$d_{ij} = 749.50$$

Exercise 38

The nearest-neighbour (single-link) clustering method

	1	2	3	4	5
1	0.00	0.02	0.06	0.10	0.09
2	0.02	0.00	0.05	0.09	0.08
3	0.06	0.05	0.00	0.04	0.05
4	0.10	0.09	0.04	0.00	0.03
5	0.09	0.08	0.05	0.03	0.00

Exercise 38



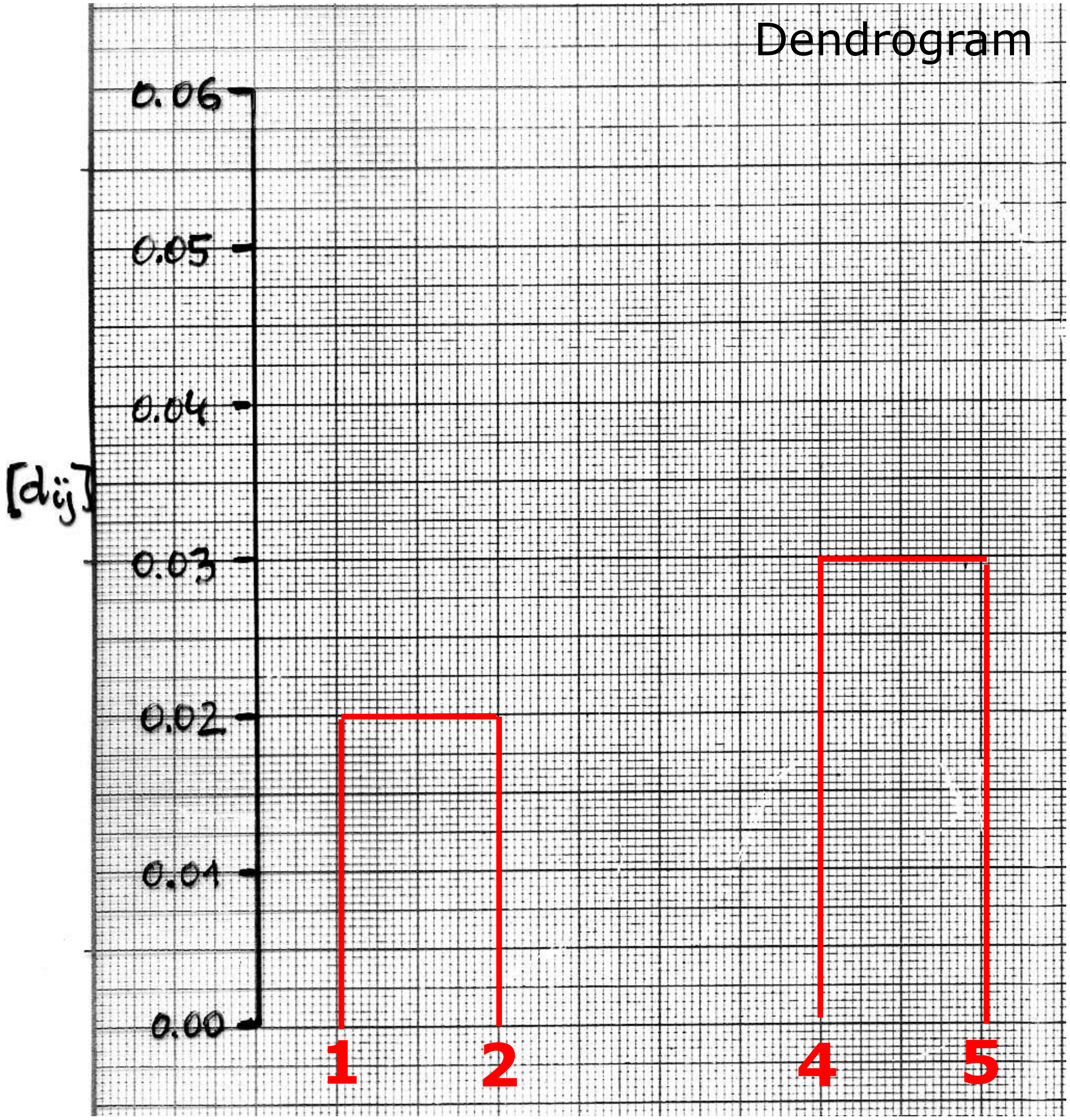
Exercise 38

		(1,2)				
		1	2	3	4	5
(1,2)	1	0.00	0.02	0.06	0.10	0.09
	2	0.02	0.00	0.05	0.09	0.08
	3	0.06	0.05	0.00	0.04	0.05
	4	0.10	0.09	0.04	0.00	0.03
	5	0.09	0.08	0.05	0.03	0.00

Exercise 38

				(4,5)	
		(1,2)	3	4	5
	(1,2)	0.00	0.05	0.09	0.08
	3	0.05	0.00	0.04	0.05
(4,5)	4	0.09	0.04	0.00	$\boxed{0.03}$
	5	0.08	0.05	$\boxed{0.03}$	0.00

Exercise 38



Exercise 38

	(1,2)	3	(4,5): 4	(4,5): 5
(1,2)	0.00	0.05	0.09	0.08
3	0.05	0.00	0.04	0.05
(4,5): 4	0.09	0.04	0.00	0.03
(4,5): 5	0.08	0.05	0.03	0.00

$d_{(12)3} = 0.05$ (as before)

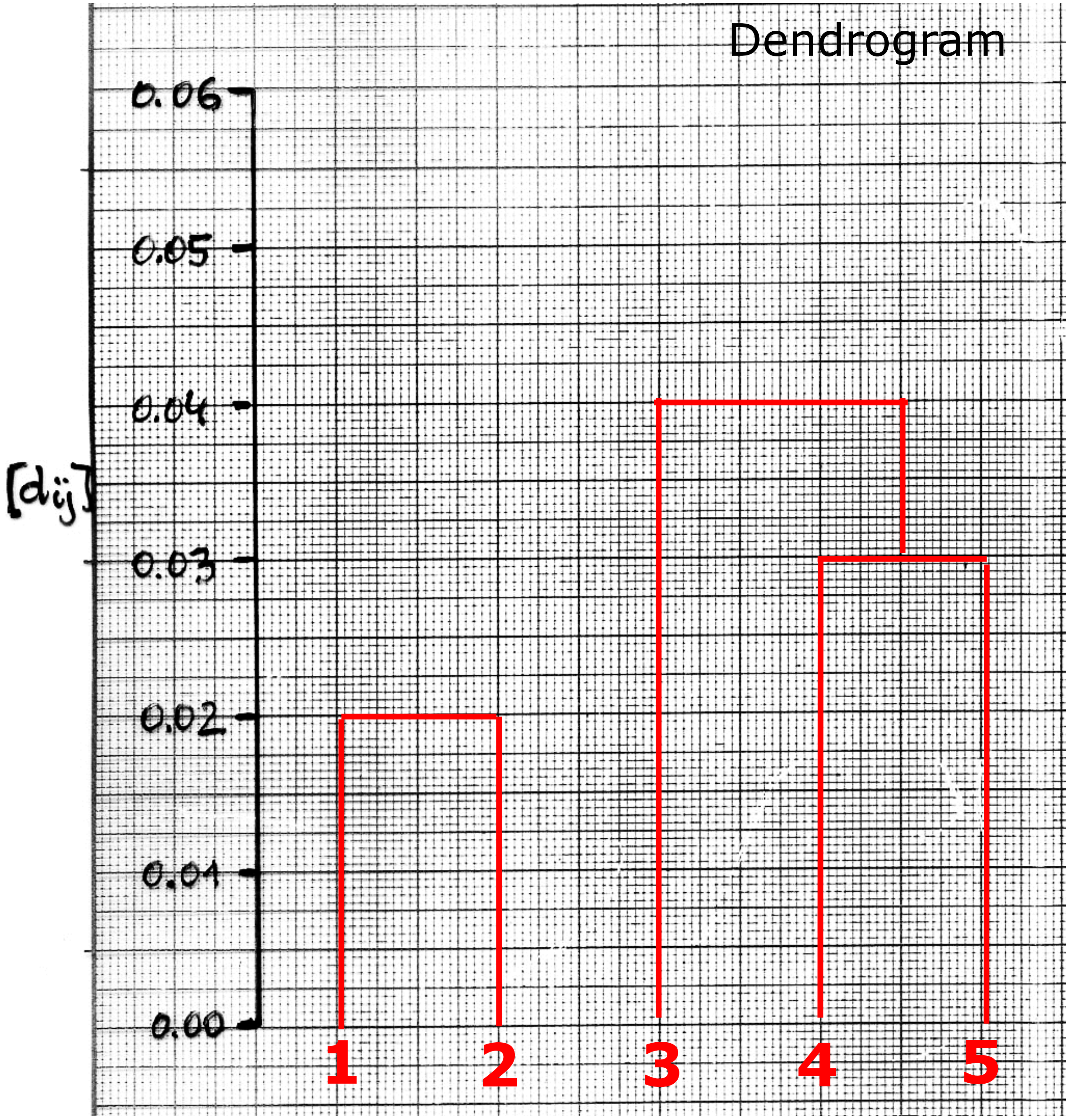
$d_{(12)(45)} = \min\{d_{14}, d_{15}, d_{24}, d_{25}\} = d_{25} = 0.08$

$d_{(45)3} = \min\{d_{34}, d_{35}\} = d_{34} = 0.04$

Exercise 38

			(3,4,5)	
		(1,2)	3	(4,5)
	(1,2)	0.00	0.05	0.08
(3,4,5)	3	0.05	0.00	$\boxed{0.04}$
	(4,5)	0.08	$\boxed{0.04}$	0.00

Exercise 38

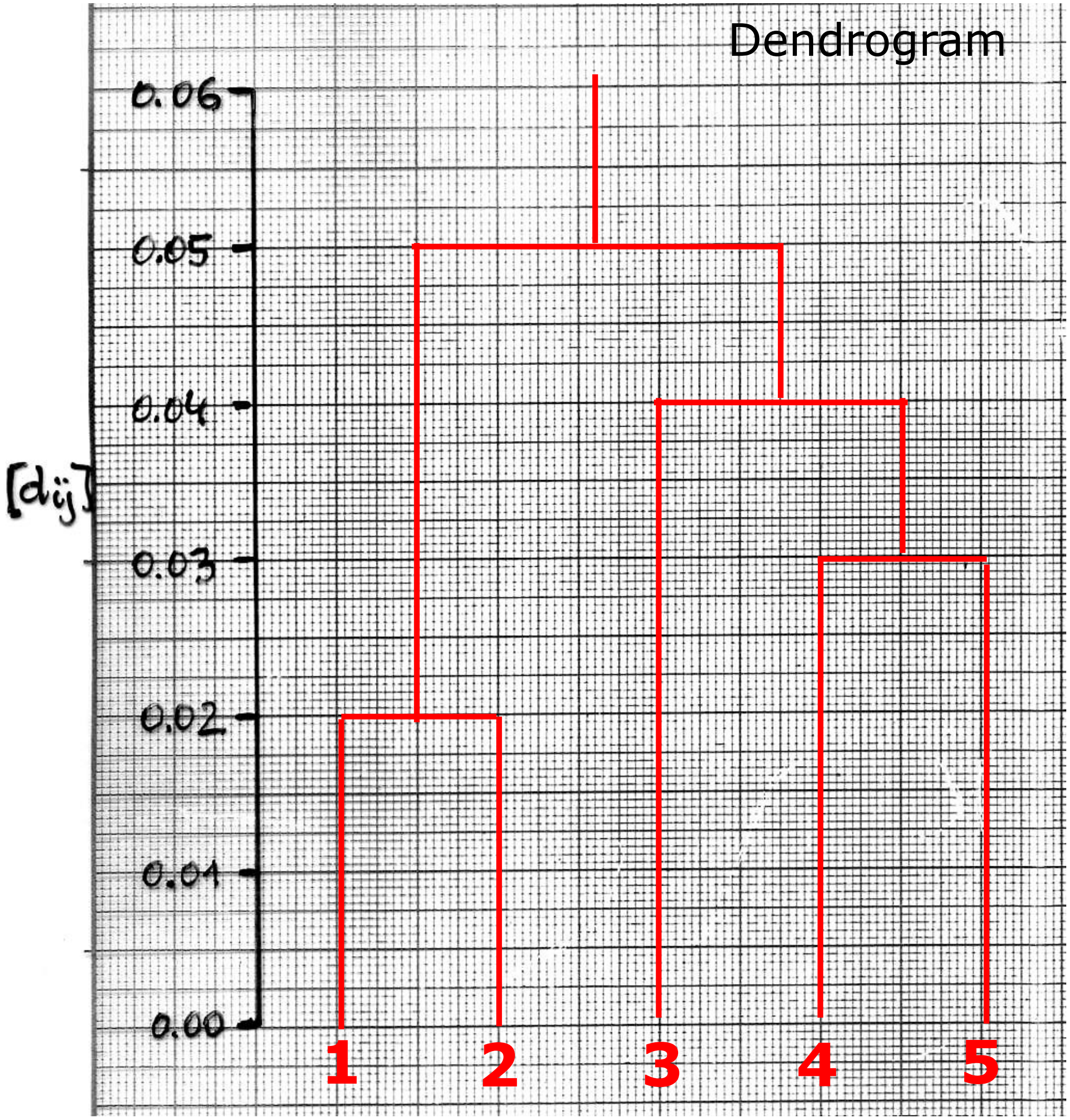


Exercise 38

			(3,4,5)	
		(1,2)	3	(4,5)
	(1,2)	0.00	0.05	0.08
(3,4,5)	3	0.05	0.00	0.04
	(4,5)	0.08	0.04	0.00

0.00	0.05
0.05	0.00

Exercise 38



Exercise 39

Pair-group method

	A	B	C	D	E	F
A	1.00	0.57	0.12	-0.65	-0.62	-0.39
B	0.57	1.00	0.46	-0.79	-0.72	-0.72
C	0.12	0.46	1.00	-0.58	-0.61	-0.52
D	-0.65	-0.79	-0.58	1.00	0.66	0.41
E	-0.62	-0.72	-0.61	0.66	1.00	0.40
F	-0.39	-0.72	-0.52	0.41	0.40	1.00

	A	B	C	D	E	F
A	1.00	0.57	0.12	-0.65	-0.62	-0.39
B	0.57	1.00	0.46	-0.79	-0.72	-0.72
C	0.12	0.46	1.00	-0.58	-0.61	-0.52
D	-0.65	-0.79	-0.58	1.00	0.66	0.41
E	-0.62	-0.72	-0.61	0.66	1.00	0.40
F	-0.39	-0.72	-0.52	0.41	0.40	1.00

$$r_{(AB)C} = \frac{0.12 + 0.46}{2} = 0.29$$

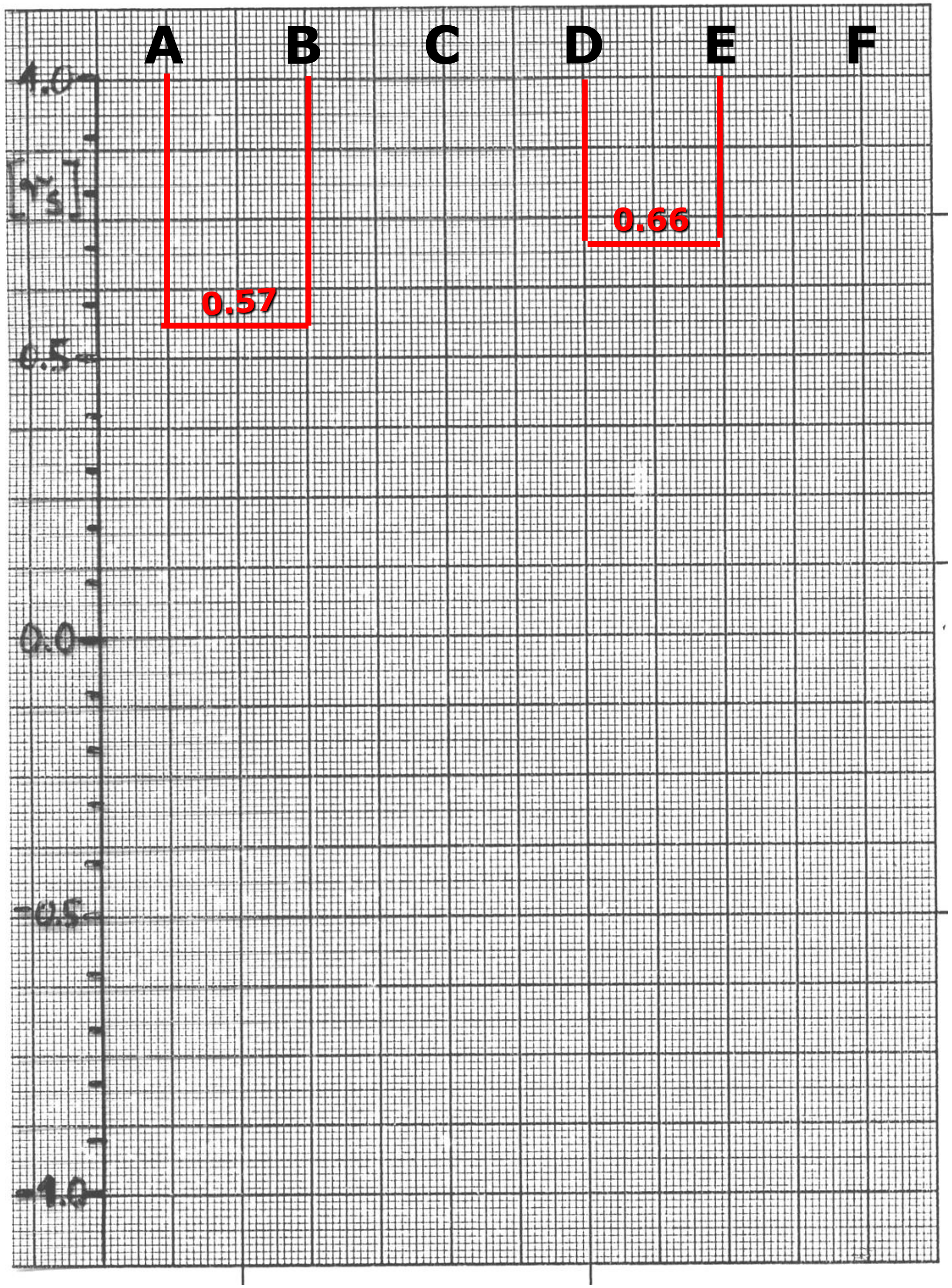
$$r_{(AB)(DE)} = \frac{(-0.65) + (-0.62) + (-0.79) + (-0.72)}{4} = -0.70$$

$$r_{(AB)F} = \frac{(-0.39) + (-0.72)}{2} = -0.55$$

$$r_{(DE)F} = \frac{0.41 + 0.40}{2} = 0.405 \simeq 0.41$$

	(AB)	C	(DE)	F
(AB)	1.00	0.29	-0.70	-0.55
C	0.29	1.00	-0.59	-0.52
(DE)	-0.70	-0.59	1.00	0.41
F	-0.55	-0.52	0.41	1.00

Exercise 39



Exercise 39

	A	B	C	D	E	F
A	1.00	0.57	0.12	-0.65	-0.62	-0.39
B	0.57	1.00	0.46	-0.79	-0.72	-0.72
C	0.12	0.46	1.00	-0.58	-0.61	-0.52
D	-0.65	-0.79	-0.58	1.00	0.66	0.41
E	-0.62	-0.72	-0.61	0.66	1.00	0.40
F	-0.39	-0.72	-0.52	0.41	0.40	1.00

$$r_{(AB)C} = \frac{0.12 + 0.46}{2} = 0.29$$

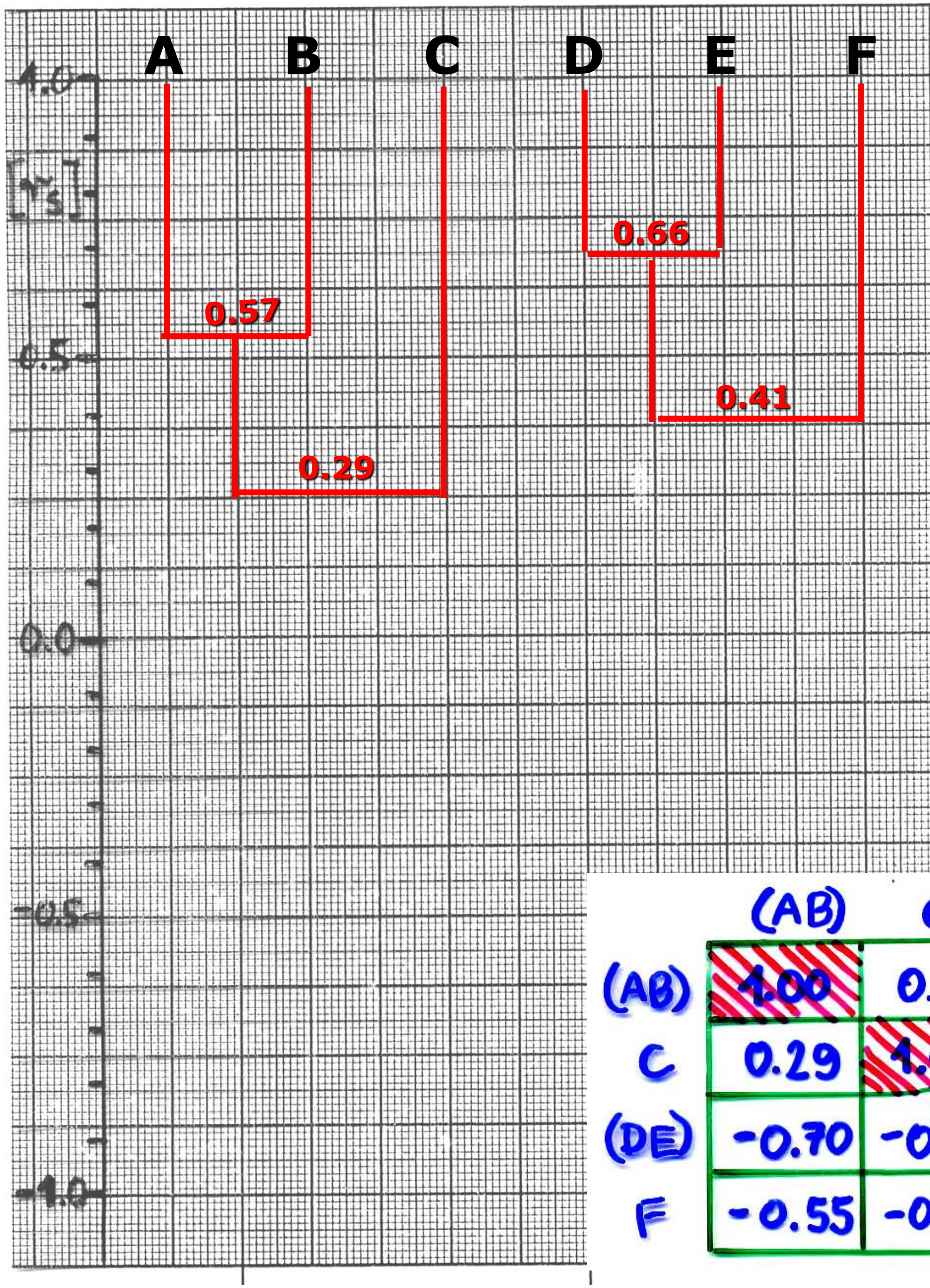
$$r_{(AB)(DE)} = \frac{(-0.65) + (-0.62) + (-0.79) + (-0.72)}{4} = -0.70$$

$$r_{(AB)F} = \frac{(-0.39) + (-0.72)}{2} = -0.55$$

$$r_{(DE)F} = \frac{0.41 + 0.40}{2} = 0.405 \simeq 0.41$$

	(AB)	C	(DE)	F
(AB)	1.00	0.29	-0.70	-0.55
C	0.29	1.00	-0.59	-0.52
(DE)	-0.70	-0.59	1.00	0.41
F	-0.55	-0.52	0.41	1.00

Exercise 39



	(AB)	C	(DE)	F
(AB)	1.00	0.29	-0.70	-0.55
C	0.29	1.00	-0.59	-0.52
(DE)	-0.70	-0.59	1.00	0.41
F	-0.55	-0.52	0.41	1.00

Exercise 39

	(AB)	C	(DE)	F
(AB)	1.00	0.29	-0.70	-0.55
C	0.29	1.00	-0.59	-0.52
(DE)	-0.70	-0.59	1.00	0.41
F	-0.55	-0.52	0.41	1.00

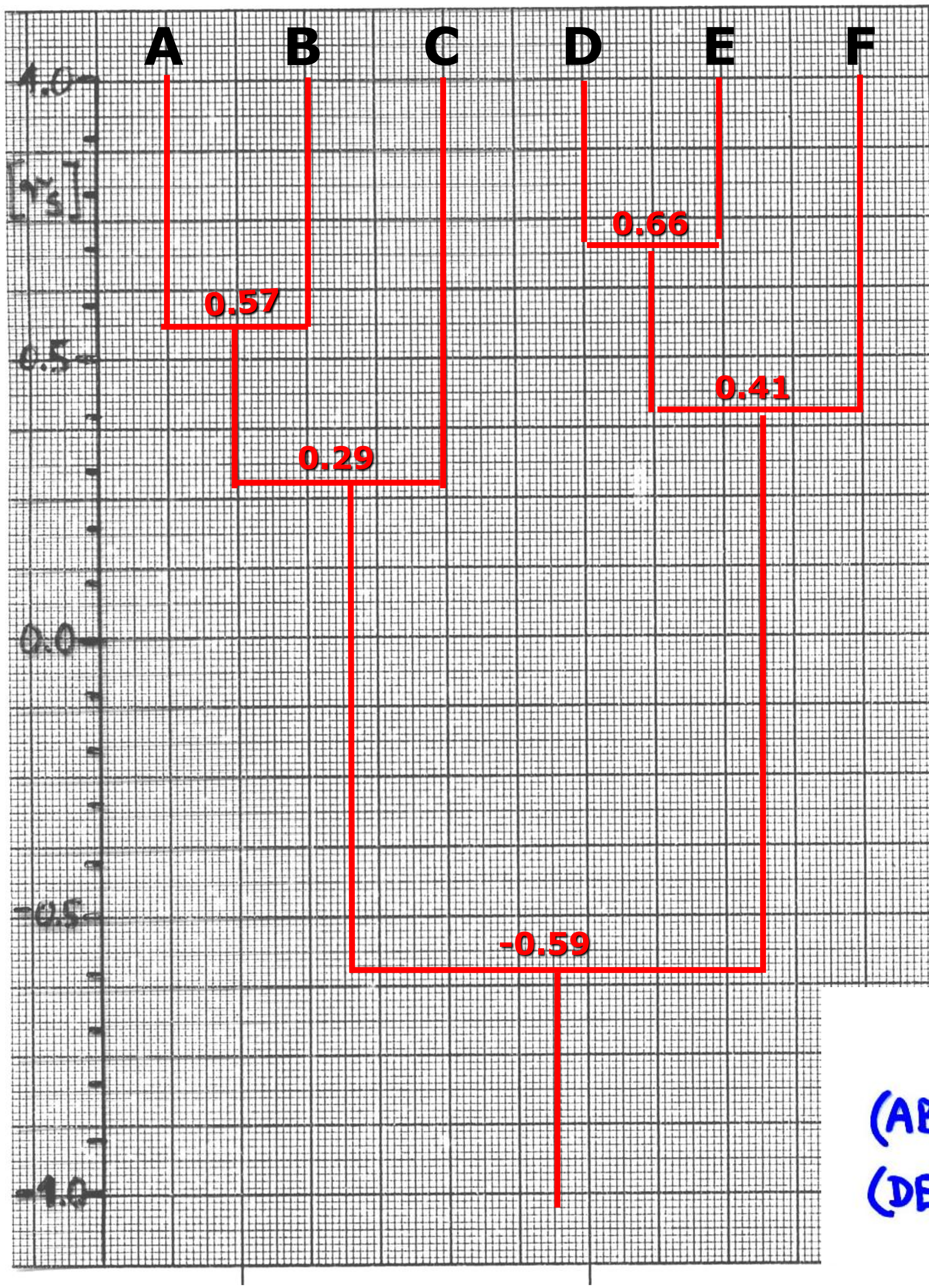
1.00

1.00

$$r_{(ABC)(DEF)} = \frac{(-0.70) + (-0.59) + (-0.55) + (-0.52)}{4} = -0.59$$

	(ABC)	(DEF)
(ABC)	1.00	[redacted]
(DEF)	[redacted]	1.00

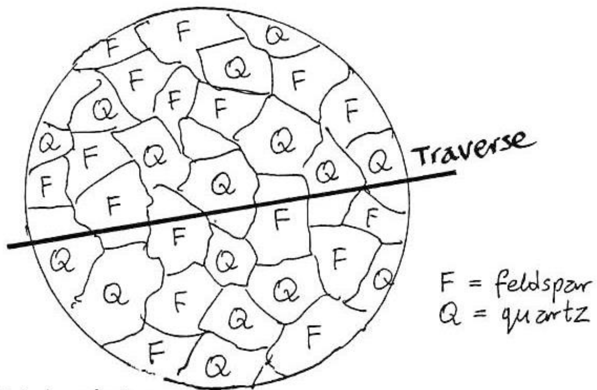
Exercise 39



	(ABC)	(DEF)
(ABC)	1.00	-0.59
(DEF)	-0.59	1.00

Exercise 40

Exercise (simple runs test, p. 94)



Sequence of 108 grains in a pegmatite

feldspar: n_1 = 70 grains
quartz: n_2 = 38 grains

Start of data series

FQQFQQFFQFQFFFFFFFQQFQFFFQFFFFQFFFQQFQFQQFFFFFQFFFFFQQQ
FFQQQFFFFFFQFQFFFFFQFQFQFFQFFFFFQFFFQQFQFFQ **FFFQFQFF**

End of series

Calculate: $U_0 =$

$\bar{u}_e =$

$S^2_{\bar{u}} =$

$S_{\bar{u}} =$

$$z = \frac{|u - \bar{u}|}{S_{\bar{u}}} =$$

$Z_{0.10} =$

Conclusion:

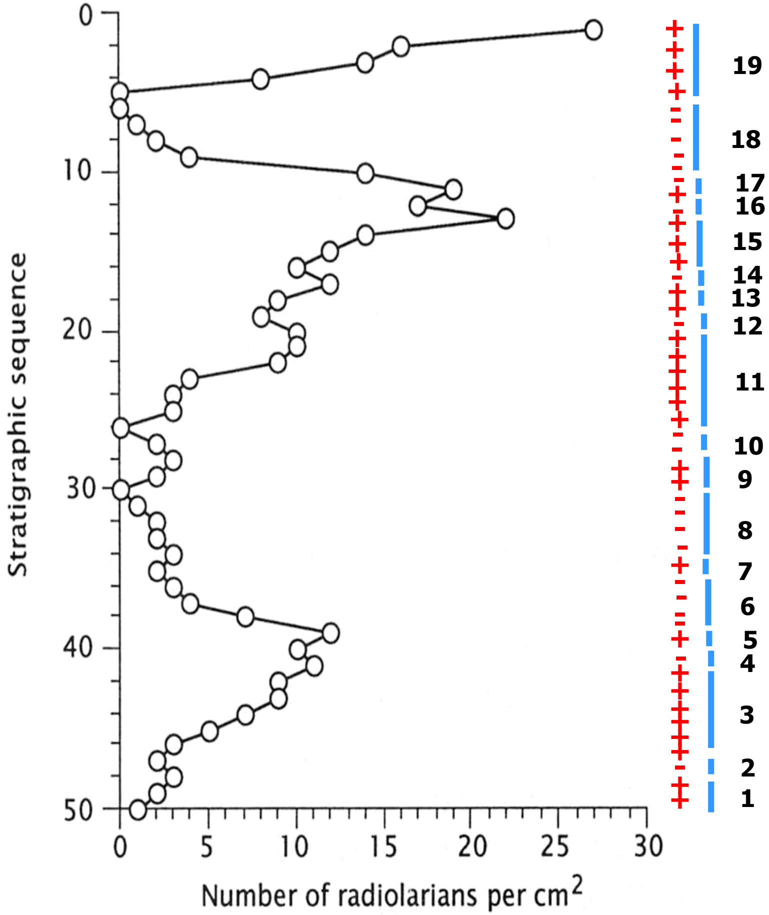
Exercise 41

Numbers of radiolarian tests per square centimeter in thin sections of siliceous shale.

(Bottom of section)	1	2	3	2	3	5	7	9	9	11	10	12	7	4	3	2	3	
	2	2	1	0	2	3	2	0	3	3	4	9	10	10	8	9	12	
	10	12	14	22	17	19	14	4	2	1	0	0	8	14	16	27		(Top of section)

Data series of 50 samples

$n_{+} = 28$, $n_{-} = 21$, $N = 49$



$U_o = 19$

$U_e = [(2 \cdot 28 \cdot 21)/49] - 1 = 23$

$$\begin{cases} H_0: U_o \geq U_e \\ H_1: U_o < U_e \end{cases}$$

$$S_U = \sqrt{[2 \cdot 28 \cdot 21\,(2 \cdot 28 \cdot 21 - 49)]/49^2(49-1)} = 3.39$$

$Z = |19 - 23|/3.39 = 1.18$

$Z_{0.10} = 1.28$

Conclusion:

H_o cannot be rejected with 90% confidence

Exercise 42

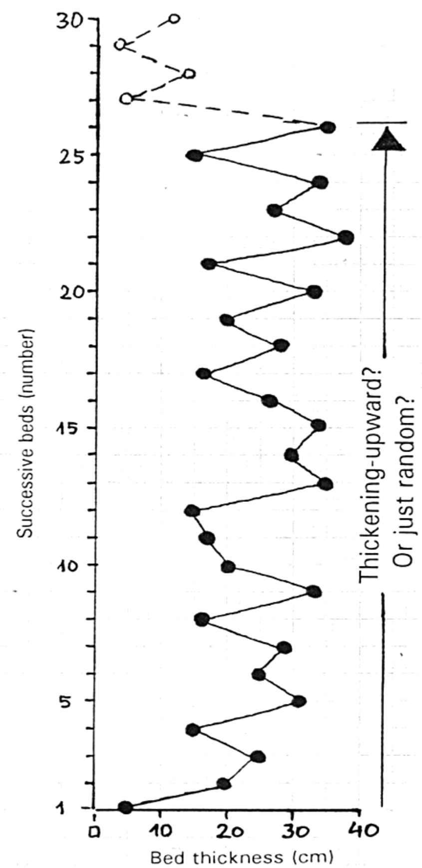
Testing of stratigraphic trends

A turbidite succession has the following bed thicknesses (in cm):

(base) **5 20 25 15 31 25 29 16 33 20 17 15 35 30 34 26 16 28 20 33 17 38 27 34 15 35** (top)

These thickness data have been plotted below (see diagram) for an easy overview.

Does this succession show a "thickening-upward" trend, or is it just random?



Testing of stratigraphic trends

Exercise 42

A turbidite succession has the following bed thicknesses (in cm):

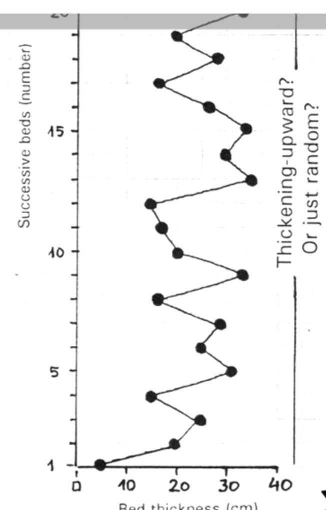
(base) 5 20 25 15 31 25 29 16 33 20 17 15 35 30 34 26 16 28 20 33 17 38 27 34 15 35 (top)

DATA:	5	20	25	15	31	25	29	16	33	20	17	15	35	30	34	26	16	28	20	33	17	38	27	34	15	35
PR:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
VR:																										
d^2:																										

$\sum d^2 = \boxed{}$

$n = \boxed{26}$

Your conclusion: ....



Calculate:

$$r_s = 1 - \frac{6\sum d^2}{n(n^2 - 1)} = \boxed{}$$

$$t = \frac{|r_s|}{\sqrt{1/(n-1)}} = \boxed{}$$

Compare the calculated t-value with the critical values of this function, t_α for d.f. = (n - 1) and a selected risk-of-error level. If $t > t_\alpha$, the data series is non-random; otherwise, it has to be regarded as random.

RANK-CORRELATION TEST

Number of ties:

3
2
2
3
2
2
2
2

18

DATA:	5	20	25	15	31	25	29	16	33	20	17	15	35	30	34	26	16	28	20	33	17	38	27	34	15	35
PR:																										
VR:																										
d^2:																										

$\sum d^2 = \boxed{}$

$n = \boxed{}$

Your conclusion: ....

Exercise 42

Calculation

If there are ties in *VR*-indices within the data series, such that one or more mean *VR*s have to be used, the formula for the Spearman correlation coefficient needs to be corrected as follows (Kendall and Gibbons, 1990):

$$\boxed{r_S = \frac{E_{PR} + E_{VR} - \Sigma d^2}{2\sqrt{E_{PR} + E_{VR}}}}$$

with: $E_{PR} = (n^3 - n)/12$ and $E_{VR} = [(n^3 - n)/12] - [(w^3 - w)/12]$, where n is the total number of data in the series and w is the number of data tied by *VR*-indices.

In the present case:

$w = 18$

$E_{PR} = (26^3 - 26)/12 = 1462.5$ and $E_{VR} = [(26^3 - 26)/12] - [(18^3 - 18)/12] = 978.0$

and

$r_S = (1462.5 + 978.0 - 1902)/2 \cdot 1194.12 = 0.2255$

$$\boxed{t = 0.2255/\sqrt{1/(26-1)} = 0.2255/0.2 = 1.127}$$

$DF = 26 - 1 = 25$

The statistical null and alternative hypotheses tested are:

$$\begin{cases} H_0: r_S \leq 0 \\ H_1: r_S > 0 \end{cases}$$

NB! This is a one-tail formulation. The risk of error will then be α and the critical value will be read off for α.

Exercise 43

Another example

$\{x\}$

	PR	VR	$D^2=(PR-VR)^2$
55.30	11	11	0
52.13	10	10	0
45.19	9	4	25
48.27	8	9	1
44.13	7	1	36
44.61	6	2	16
44.85	5	3	4
46.90	4	7	9
45.65	3	5	4
48.15	2	8	36
46.37	1	6	25
			$\Sigma D^2 = 156$

$n=11$

$$r_s = 1 - \frac{6 \cdot 156}{11(11^2-1)} = 0.291$$

$$\begin{cases} H_0: & \rho_s \leq 0 \\ H_1: & \rho_s > 0 \end{cases}$$

Test function (p. 69):

$$t = 0.291\sqrt{\frac{11-2}{1-0.291^2}} = 0.912$$

$DF = 9$

$\alpha = 0.05$ $\quad t_{0.05} =$

$\alpha = 0.10$ $\quad t_{0.10} =$

Conclusion:

Exercise 43

$n = 11$

D^2	VR	PR	$\{x\}$	PR	VR	D^2
0	1	1	0.21	11	1	100
4	4	2	1.11	10	4	36
0	3	3	0.43	9	3	36
1	5	4	1.21	8	5	9
9	2	5	0.25	7	2	25
0	6	6	1.71	6	6	0
0	7	7	5.54	5	7	4
0	8	8	5.80	4	8	16
1	10	9	6.35	3	10	49
1	9	10	5.25	2	9	49
0	11	11	9.61	1	11	100

$\Sigma D^2 = 16$

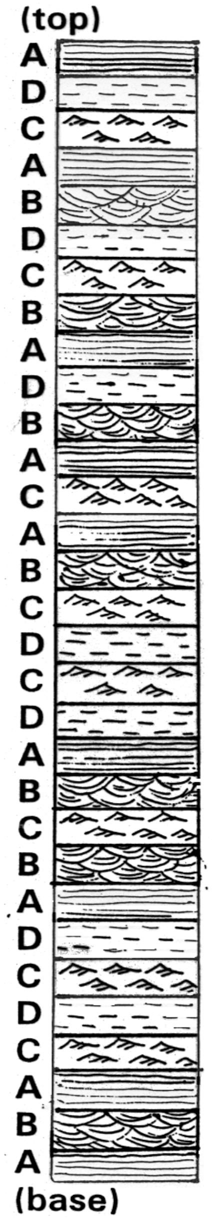
$\Sigma D^2 = 424$

$$r_s = 1 - \frac{6 \cdot 16}{1320} = 0.927$$

$$r_s = 1 - \frac{6 \cdot 424}{1320} = -0.927$$

FACIES SUCCESSION

Exercise 44



Facies:

- A = sandstone with plane-parallel stratification
- B = sandstone with large-scale trough cross-stratification
- C = sandstone with current-ripple cross-lamination
- D = mudstone

$[n_{ij}]$ row sums $= n_i$

	A	B	C	D	
A					
B					
C					
D					

$N =$

$[p_{ij}]$

	A	B	C	D	
A					=1
B					=1
C					=1
D					=1

$[e_{ij}]$

	A	B	C	D
A				
B				
C				
D				

$[d_{ij}]$

	A	B	C	D
A				
B				
C				
D				

$\left[\frac{(n_{ij}-n_i e_{ij})^2}{n_i e_{ij}}\right]$

	A	B	C	D	row sums
A					=
B					=
C					=
D					=

DF = $(4-1)^2 = 9$

d.f. = $\boxed{9}$

Critical value (for $\alpha = \boxed{}$ %): $\chi^2_{\frac{1}{2}\alpha} = \boxed{}$

$\chi^2 = \boxed{}$